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Corrigendum to: The damage number of the Cartesian product of graphs

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Abstract

In a recent paper [M.A. Huggan, M.E. Messinger, A. Porter, The damage number of the Cartesian product of graphs, *Australas. J. Combin.* 88 (2024), 362–384], the authors used an incorrect definition which then had implications to a few new theorems. In this corrigendum, we note the appropriate definition, and how it affects the statements, proofs, and a few discussion items.

The terms o-dominate and c-dominate, as defined below, were used throughout [4].

Definition 1 ([2]). A vertex u of a graph G is *o-dominated* if there exists a vertex $v \in V(G)$ such that $N(u) \subseteq N(v)$. A vertex u of a graph G is *c-dominated* if there exists a vertex $v \in V(G)$ such that $N[u] \subseteq N[v]$.

In [4], the following definition of *dominated* should have been used instead of o-dominated in several instances which we highlight below.

Definition 2 ([3]). A vertex u of a graph G is *dominated* if there exists a vertex $v \in V(G)$ such that $N(u) \subseteq N[v]$. In such a case, we say that v dominates u.

On page 372 of [4], we discuss the relationship between dominated vertices in trees T and T'; and dominated vertices in the Cartesian product. It should read that if a vertex occupied by the cop dominates a vertex occupied by the robber in T and T', then in the Cartesian product, $T \square T'$, the cop will o-dominate the vertex occupied by the robber.

In the following theorem, the change is from s o-dominating w, to s dominating w.

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Theorem 3.5. *For a graph G,* $\mathrm{dmg}(G) = 1$ *if and only if* $\mathrm{rad}(G) = 2$ *and a center of the graph $c \in V(G)$ is such that for all $w \in V(G) \setminus N[c]$ there exists $s \in N[c]$ such that s dominates w.*

In the proof of Theorem 3.5, the neighbourhood of s should be closed. In symbols, $N(s)$ should be $N[s]$. Any mention of o-dominates, becomes dominates.

In the discussion immediately after the proof of Theorem 3.5, we consider a situation where a vertex s o-dominates w, where in fact, it should be that s dominates w. We note that for the class of graphs with cop number 2, a characterization for damage number 1 graphs was given by Carlson et al. [1] in the context of throttling.

The changes to Theorem 3.9 are that G cannot have damage number 1, which previously was implicitly assumed, and condition 2 changed from o-dominates to dominates. Again, in the proof, the only change is o-dominates becomes dominates.

Theorem 3.9. *Let G be a graph with* $\mathrm{rad}(G) = 2$ *or* $\mathrm{rad}(G) = 3$*, and* $\mathrm{dmg}(G) \neq 1$*. Then* $\mathrm{dmg}(G) = 2$ *if and only if there exist vertices $z, y \in V(G)$ and $s_y \in N[z]$ such that*

1. $\mathrm{dist}_G(z, y) \in \{2, 3\}$*, and*
2. *no vertex in $N[z]$ dominates y, and*
3. $\forall\ x \in N(y)\backslash N[s_y]$*,* $\exists\ s_x \in N[s_y]$ *and* $v \in N[s_x]$ *such that*

$$N(x)\backslash\{y\} \subseteq N[s_x] \text{ and } N(y)\backslash\{x\} \subseteq N[v];$$

and for all $w \in V(G)\backslash\{y\}$ such that $\mathrm{dist}_G(z, w) \in \{2, 3\}$*, the above three conditions apply; or the conditions for* $\mathrm{dmg}(G) = 1$ *apply.*

References

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[2] N. E. Clarke and R. J. Nowakowski, Tandem-win graphs, *Discrete Math.* 299 (2005), 56–64.

[3] E. Dahlhaus, P. Hammer, F. Maffray and S. Olariu, On domination elimination orderings and domination graphs, In: *Graph-Theoretic Concepts in Computer Science*, (Eds.: E.W. Mayr, G. Schmidt and G. Tinhofer), pp. 81–92, Berlin, Heidelberg (1995), Springer Berlin Heidelberg.

[4] M. A. Huggan, M. E. Messinger and A. Porter, The damage number of the Cartesian product of graphs, *Australas. J. Combin.* 88(3) (2024), 362–384.

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