AUSTRALASIAN JOURNAL OF COMBINATORICS
Volume **75(1)** (2019), Pages 171–173

Corrigendum: Trees and n-good hypergraphs

Mark Budden Andrew Penland

Department of Mathematics and Computer Science
Western Carolina University
Cullowhee, NC 28723
U.S.A.
mrbudden@email.wcu.edu adpenland@email.wcu.edu

Abstract

In a recent paper by the authors, it was incorrectly claimed that the disjoint union of multiple copies of a connected n-good hypergraph is itself n-good. This note serves to rectify this error by providing a correct proof of the evaluation of $R(aH, K_n^{(r)}; r)$.

This note is intended to serve as a corrigendum for Theorem 3.10 of [2], which is incorrect as written. We will follow the definitions and notations introduced in [2]. If H is an r-uniform hypergraph, then the weak chromatic number $\chi_w(H)$ is the least number of colors needed to color the vertices of H so that no hyperedge is monochromatic. The chromatic surplus $t(H)$ is the minimum cardinality of a color class in any weak proper vertex coloring of H that uses $\chi_w(H)$ colors. Recall that if H is an r-uniform hypergraph such that $c(H) \geq t(K_n^{(r)})$, then the following inequality holds:

$$R(H, K_n^{(r)}; r) \geq (c(H) - 1)(\chi_w(K_n^{(r)}) - 1) + t(K_n^{(r)}),$$

where $c(H)$ is the order of a maximal connected component of H (see Theorem 3.1 of [2]). When this inequality is tight,

$$R(H, K_n^{(r)}; r) = (c(H) - 1)(\chi_w(K_n^{(r)}) - 1) + t(K_n^{(r)}),$$

we say that H is n-good.

In Theorem 3.10 of [2], it was claimed that if H is a connected n-good hypergraph, then so is aH (the disjoint union of a copies of H). This claim is false, and the mistake lies in inequality (5), where we inadvertently used the order of aH instead of $c(aH)$ (i.e., replacing am with m corrects this inequality, but makes the remainder of the proof invalid). The following theorem gives a correct evaluation of $R(aH, K_n^{(r)}; r)$ when H is n-good. The proof follows the general approach used in Lemma B of Bielak's paper [1].

ISSN: 2202-3518

Theorem 1. *If H is a connected n-good r-uniform hypergraph of order $m \geq r$ and $a \geq 1$, then*

$$R(aH, K_n^{(r)}; r) = (m-1)(\chi_w(K_n^{(r)}) - 2) + am + t(K_n^{(r)}) - 1.$$

Proof. Observe that $m \geq t(K_n^{(r)}) + 1$, since $t(K_n^{(r)}) \leq r - 1$. It follows that

$$K := K_{am-1}^{(r)} \cup (\chi_w(K_n^{(r)}) - 2)K_{m-1}^{(r)} \cup K_{t(K_n^{(r)})-1}^{(r)}$$

does not contain aH as a subhypergraph. This is due to the fact that no component of aH can be contained in any copy of $K_{m-1}^{(r)}$ or $K_{t(K_n^{(r)})-1}^{(r)}$, and there are not enough vertices in $K_{am-1}^{(r)}$ to contain all of aH. To see that $\overline{K}$ does not contain a subhypergraph isomorphic to $K_n^{(r)}$, we consider two cases. First, if $t(K_n^{(r)}) = 1$, then $\chi_w(\overline{K}) = \chi_w(K_n^{(r)}) - 1$ since we can properly color $\overline{K}$ using $\chi_w(K_n^{(r)}) - 1$ colors. If $t(K_n^{(r)}) > 1$, then $\chi_w(\overline{K}) = \chi_w(K_n^{(r)})$ and $t(\overline{K}) = t(K_n^{(r)}) - 1$ since we can produce a weak proper coloring of $\overline{K}$ using $\chi_w(K_n^{(r)})$ colors that has chromatic surplus less than or equal to $t(K_n^{(r)}) - 1$. Hence, we have shown that

$$R(aH, K_n^{(r)}; r) \geq (m-1)(\chi_w(K_n^{(r)}) - 2) + am + t(K_n^{(r)}) - 1.$$

To prove the other direction, we proceed by induction on a. The case $a = 1$ follows from the assumption that the components of aH are assumed to be n-good. Assume that the theorem holds for $(a-1)H$:

$$R((a-1)H, K_n^{(r)}; r) = (m-1)(\chi_w(K_n^{(r)}) - 2) + (a-1)m + t(K_n^{(r)}) - 1.$$

Consider a red/blue coloring of $K_{(m-1)(\chi_w(K_n^{(r)})-2)+am+t(K_n^{(r)})-1}^{(r)}$ that does not contain a blue subhypergraph isomorphic to $K_n^{(r)}$ and denote by L the subhypergraph spanned by the red hyperedges. Since

$$\begin{aligned} |V(L)| &= (m-1)(\chi_w(K_n^{(r)}) - 2) + am + t(K_n^{(r)}) - 1 \\ &> (m-1)(\chi_w(K_n^{(r)}) - 2) + m + t(K_n^{(r)}) - 1, \end{aligned}$$

it follows that L contains a red subhypergraph isomorphic to H. Also note that

$$|V(L) - V(H)| = (m-1)(\chi_w(K_n^{(r)}) - 2) + (a-1)m + t(K_n^{(r)}) - 1.$$

By the inductive hypothesis, $L - H$ contains $(a-1)H$, and hence, L contains aH. Thus, we find that

$$R(aH, K_n^{(r)}; r) \leq (m-1)(\chi_w(K_n^{(r)}) - 2) + am + t(K_n^{(r)}) - 1,$$

completing the proof of the theorem. □

In particular, note that $R(aH, K_n^{(r)}; r)$ is n-good if and only if

$$(m-1)(\chi_w(K_n^{(r)}) - 2) + am + t(K_n^{(r)}) - 1 = (m-1)(\chi_w(K_n^{(r)}) - 1) + t(K_n^{(r)}),$$

which occurs if and only if $a = 1$.

References

[1] H. Bielak, Ramsey numbers for a disjoint union of good graphs, *Discrete Math.* **310** (2010), 1501–1505.

[2] M. Budden and A. Penland, Trees and n-good hypergraphs, *Australas. J. Combin.* **72**(2) (2018), 329–349.

(Received 20 Mar 2019; revised 18 June 2019)