Partitioning sets of blocks into designs

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ABSTRACT

Earlier results on partitioning sets of blocks into designs are reconsidered and extended, and interesting properties of some of these partitions are discussed.

1. Introduction

A *t–design* based on a v–set, X, is a collection of k–subsets (blocks) chosen from X in such a way that each unordered t–subset of X occurs in precisely λ of the blocks. Such a design has parameters t–(v,k,λ). In particular, for v even, a one–factor of the complete graph K_v may be regarded as a 1–$(v,2,1)$ design. Two t–(v,k,λ) designs are said to be *disjoint* if and only if they have no block in common.

If the set of all the $\binom{v}{k}$ k–sets contained in X can be partitioned into mutually disjoint t–(v,k,λ) designs (all with the same parameters), then these designs are said to form a *large set*, denoted by $LS(t$–$(v,k,\lambda))$. In particular, for v even, a one–factorisation of K_v may be regarded as a $LS(1$–$(v,2,1))$; it is also often denoted by $OF(K_v)$. If a t–(v,k,λ) design has b blocks, then b must divide $\binom{v}{k}$ for a large set of these designs to exist. However, even where this condition is satisfied a large set may not exist; for example, there is no $LS(2$–$(7,3,1))$ [4].

Whether or not a large set exists, it may be possible to pack the designs neatly by enlarging the set of points on which they are based, sometimes by adjoining just one extra point. Thus, if the set of all the $\binom{v}{k}$ k–sets chosen from X can be partitioned into v mutually disjoint t–$(v-1,k,\lambda)$ designs, each missing a different point of X, then these designs are said to form an *overlarge set*, denoted by $OS(t$–$(v-1,k,\lambda))$. We shall label the designs of an overlarge set by their missing elements. In particular, for v odd, a near–one–factorisation of K_v may be regarded as an $OS(1$–$(v-1,2,1))$; it is also often denoted by $NOF(K_v)$. If a t–$(v-1,k,\lambda)$ design has b blocks, then b must divide $\binom{v}{k}$ for an overlarge set of these designs to exist. However, even where this condition is satisfied an overlarge set may not exist; for example, there is no $OS(5$–$(12,6,1))$ [7].

There are two general constructions for overlarge sets, as follows:

(i) From any t–$(v,k,1)$ design, D, we can form an $OS((t-1)$–$(v-1,k-1,1))$ by choosing, for each $i = 1, \ldots, v$, all the blocks of D containing i, and deleting i from each of them. These $(k-1)$–sets form design $\mathcal{D}_i$, and this overlarge set is said to be *derived* from D. Note that, for different values of i, the designs $\mathcal{D}_i$ derived from D need not be isomorphic to each other. (This use of the term 'derived' is consistent with that of Rosa [6].)

(ii) From any $OS(t$–$(v,k,1))$ based on the set $\{0,1,\ldots,v\}$ and consisting of the designs $\mathcal{D}_0, \mathcal{D}_1, \ldots, \mathcal{D}_v$, we can form $v+1$ distinct $OS((t-1)$–$(v-1,k-1,1))$. Starting from some fixed element i, we first discard the design $\mathcal{D}_i$ and then, from each design $\mathcal{D}_j$, $j \neq i$, we choose all the blocks containing i, and delete i from each of them to form the design $\mathcal{E}_j$ on the set $\{0,1,\ldots,v\} \setminus \{i,j\}$. These designs $\mathcal{E}_j$, $j \neq i$, form an $OS((t-1)$–$(v-1,k-1,1))$ based on the set $\{0,1,\ldots,v\} \setminus \{i\}$; this is a *contraction* of the original overlarge set. The isomorphism class of the contraction will in general depend on the element i which has been deleted. We also refer to the original overlarge set of designs $\mathcal{D}_i$, $i = 0,1,\ldots,v$, as an *extension* of the overlarge set of designs $\mathcal{E}_j$. Note that, for $t \geq 2$, every overlarge set of t–designs is an extension of at least one overlarge set of $(t-1)$–designs, but that not every overlarge set of $(t-1)$–designs is a contraction of an overlarge set of t–designs.

Construction (i) above shows that, for example, there is an $OS(2$–$(v,3,1))$ for every $v \equiv 1$ or $3 \pmod 6$, since there is a 3–$(v+1,4,1)$ design for every such v [3]. But this is not the only way in which such overlarge sets arise. For some small designs, we know all possible overlarge sets and, consequently, we know which of them are extendible; this information is summarised in Table 1.

Construction (ii) for overlarge sets has an analogue for large sets; that is, from a $LS(t$–$(v,k,1))$ we can form a $LS((t-1)$–$(v-1,k-1,1))$ by contraction.

In this paper, we consider the following:

Parameters of designs	Number of non–isomorphic overlarge sets	Number of extendible overlarge sets
1–(6,2,1)	7	2
2–(7,3,1)	11 [9]	2 [1]
3–(8,4,1)	2 [1]	0
1–(8,2,1)	3460	150
2–(9,3,1)	77 [10]	40 [8]
3–(10,4,1)	21 [8]	21 [7]
4–(11,5,1)	24 [7]	0 [7]

Table 1: The number of non–isomorphic overlarge sets of some small t–designs, and the number of these overlarge sets which are extendible.

(i) relationships between $LS(2\text{–}(9,3,1))$ and their contractions to $OF(K_8)$;

(ii) relationships between $OS(2\text{–}(7,3,1))$, their contractions to $NOF(K_7)$ and their extensions to $OS(3\text{–}(8,4,1))$;

(iii) relationships between $OS(2\text{–}(9,3,1))$, their contractions to $NOF(K_9)$, their extensions to $OS(3\text{–}(10,4,1))$ and further to $OS(4\text{–}(11,5,1))$;

(iv) the possibility of using contraction and extension of a given large or overlarge set of designs to define a coarser equivalence relation, more appropriate than isomorphism, in cases where the number of isomorphism classes becomes very large.

2. Contractions of LS(2–(9,3,1))

There are precisely two $LS(2\text{–}(9,3,1))$ [4], and precisely six $OF(K_8)$ [11]; these are listed for convenience in Tables 2 and 3 respectively. Note that from each 3×3 array in Table 2, the corresponding 2–(9,3,1) design is constructed as usual by taking the rows, columns, forward diagonals and back diagonals of the array of the design. In Table 3, G_j denotes the automorphism group of $\mathfrak{F}_j$. The groups G_2, G_3 and G_4 are all subgroups of G_1; G_5 is isomorphic to the subgroup of G_1 with generators (124)(678), (132)(587) and (17)(28)(35)(46); G_6 is not isomorphic to any subgroup of G_1. Contraction of the first large set on element 1 (or 5) leads to a one–factorisation isomorphic to $\mathfrak{F}_1$ and contraction on any other element to a one–factorisation isomorphic to $\mathfrak{F}_5$. Contraction of the second large set on any element also leads to a one–factorisation isomorphic to $\mathfrak{F}_5$. The relationships between these designs are shown in Figure 1.

There is no $LS(3\text{–}(10,4,1))$ [4], so these large sets cannot be extended.

3. Contractions and extensions of OS(2–(7,3,1))

From the six $OF(K_8)$ listed in Table 3 we can obtain the seven $NOF(K_7)$ listed in Table 4. In this table, a contraction of the $OF(K_8)$ $\mathfrak{F}_j$ on the point i is labelled $\mathfrak{N}_{ji}$

Large Set	Designs						
1	139 275 486	192 745 863	127 485 639	174 865 392	148 635 927	186 395 274	163 925 748
2	124 378 956	128 943 765	125 983 476	129 743 586	123 469 785	126 357 489	127 346 598

Table 2: The two large sets of 2–(9,3,1) designs.

OF ; $\mid G_j \mid$; G_j Generators	Blocks	OF ; $\mid G_j \mid$; G_j Generators	Blocks
$\mathfrak{F}_1$; 1344 ; $AGL(3,2)$ (0561732) (45)(67)	01 23 45 67 02 13 46 57 03 12 47 56 04 15 26 37 05 14 27 36 06 17 24 35 07 16 25 34	$\mathfrak{F}_2$; 64 ; $\mathbb{Z}_2^4 \cdot \mathbb{Z}_2^2$ (02)(13)(46)(57) (24)(35) (45)(67)	01 23 45 67 02 13 46 57 03 12 47 56 04 15 26 37 05 14 27 36 06 17 25 34 07 16 24 35
$\mathfrak{F}_3$; 16 ; $D_8 \times \mathbb{Z}_2$ (01)(23) (02)(13)(45)(67) (0614)(2735)	01 23 45 67 02 13 46 57 03 12 47 56 04 16 25 37 05 17 26 34 06 14 27 35 07 15 24 36	$\mathfrak{F}_4$; 96 ; $\mathbb{Z}_2^4 \cdot S_3$ (0415)(2736) (123)(567)	01 23 45 67 02 13 46 57 03 12 47 56 04 16 27 35 05 17 26 34 06 14 25 37 07 15 24 36
$\mathfrak{F}_5$; 24 ; $A_4 \times \mathbb{Z}_2$ (01)(23)(46)(57) (135)(267)	01 23 45 67 02 13 46 57 03 14 27 56 04 16 25 37 05 17 26 34 06 12 35 47 07 15 24 36	$\mathfrak{F}_6$; 42 ; $\mathbb{Z}_7 \cdot \mathbb{Z}_6$ (01)(45)(67) (153)(476)	01 23 45 67 02 14 36 57 03 16 25 47 04 17 26 35 05 12 37 46 06 15 27 34 07 13 24 56

Table 3: The six non-isomorphic one-factorizations of K_8. G_j denotes the automorphism group of the one-factorization of $\mathfrak{F}_j$. For two groups H and L, $H \times L$ denotes their direct product and $H \cdot L$ denotes their semi-direct product. D_8, A_4, S_3 and $\mathbb{Z}_n$ denote the dihedral group of order 8, the alternating group of degree 4, the symmetric group of degree 3 and the cyclic group of order n respectively. $\mathbb{Z}_n^m$ denotes the direct product of m copies of $\mathbb{Z}_n$. $AGL(3,2)$ is the collineation group of the affine 3-space over $GF[2]$.

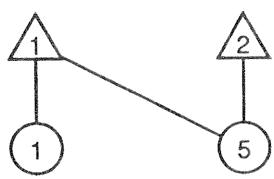

Figure 1: The relationships between the two large sets of 2-(9,3,1) designs (denoted by triangles) and the two extendible one-factorizations, $\mathfrak{F}_1$ and $\mathfrak{F}_5$, of K_8 (denoted by circles).

and has automorphism group H_{ji}, which is the stabiliser in G_j of the point i. The groups of all of the one-factorisations $\mathfrak{F}_j$, $1 \le j \le 5$, are transitive on the points of the design, so just one near-one-factorisation, $\mathfrak{N}_{j0}$, arises from each. However the group of the 1-rotational one-factorisation $\mathfrak{F}_6$ fixes one point and is transitive on the rest; it gives rise to the two near-one-factorisations, $\mathfrak{N}_{60}$ and $\mathfrak{N}_{62}$.

Each of these $NOF(K_7)$ can be extended to an OS(2–(7,3,1)); the relationships between them are shown in Figure 2. The relationships between the OS(2–(7,3,1)) and OS(3–(8,4,1)) are also shown in Figure 2: only two of the 11 OS(2–(7,3,1)) can be extended, type B going to L_1, the OS(3–(8,4,1)) with automorphism group of order 1512, and type F to L_2 with group of order 216. Note that only the $NOF(K_7)$ arising from $\mathfrak{F}_6$ lead to OS(3–(8,4,1)).

Since there is no 4–(9,5,1) design, no OS(3–(8,4,1)) is extendible.

4. Contractions and extensions of OS(2–(9,3,1))

The $OF(K_{10})$ have been classified by Gelling [2]; there are precisely 396 of them. These give rise to 3460 $NOF(K_9)$, only 150 of which can be extended to OS(2–(9,3,1)). Table A1 (in the appendix) lists these 150 $NOF(K_9)$; again, if a particular $NOF(K_9)$ was formed by contraction, on i, of the jth $OF(K_{10})$ in Gelling's list, then the value of j is called the *index* of the $NOF(K_9)$ and both the index and i are listed in the table. Also the automorphism group of the $NOF(K_9)$ is denoted by G, and the table lists both the order and the generators of G.

Figure A1 shows the relationships between these $NOF(K_9)$ and the OS(2–(9,3,1)). This graph has 11 components, the largest of which (Component 1) contains the overlarge set derived from a 3–(10,4,1) design. In fact, it contains 44 of the OS(2–(9,3,1)), including all forty that are extendible and four which are not (numbers 18, 54, 55, 58). Note that for this case, because of the difficulty of using our previous notation to label the corresponding figures (A1) we have simply labelled the $NOF(K_9)$ from 1 to 150.

For OS(2–(9,3,1)), OS(3–(10,4,1)) and OS(4–(11,5,1)), the relationships are summarised in Figures A2 and A3. Each of these graphs showing the extension–contraction relationships has three components, one of which includes almost all the overlarge sets and contains that derived from a 3–(10,4,1) or 4–(11,5,1) design respectively.

Of the 40 extendible OS(2–(9,3,1)), 16 extend in only one way each (numbers 14, 15, 21, 22, 32, 39, 46, 49, 65, 66, 69, 70, 73, 74, 75, 76), six extend in only two ways each (numbers 2, 8, 23, 24, 59, 77), twelve in three ways each (numbers 1, 5, 9, 10, 11, 16, 17, 25, 29, 30, 31, 50), four in four ways each (numbers 3, 4, 7, 12), one in five ways (number 6) and one in six ways (number 13).

All of the 21 OS(3–(10,4,1)) are extendible: two of them extend in only one way each (numbers 17, 18), ten in only two ways each (numbers 4, 11, 12, 13, 14, 15, 16, 19, 20, 21), five in three ways each (numbers 2, 5, 6, 8, 10) and four in four ways each (numbers 1, 3, 7, 9).

NOF ; j ; i ; $\mid H_{ji} \mid$; generators	Blocks	NOF ; j ; i ; $\mid H_{ji} \mid$; generators	Blocks
$\mathfrak{N}_{10}$; 1 ; 0 ; 168 (124)(365) (45)(67)	12 47 56 13 46 57 14 27 36 15 26 37 16 25 34 17 24 35 23 45 67	$\mathfrak{N}_{20}$; 2 ; 0 ; 8 (24)(35) (45)(67)	12 47 56 13 46 57 14 27 36 15 26 37 16 24 35 17 25 34 23 45 67
$\mathfrak{N}_{30}$; 3 ; 0 ; 2 (46)(57)	12 47 56 13 46 57 14 27 35 15 24 36 16 25 37 17 26 34 23 45 67	$\mathfrak{N}_{40}$; 4 ; 0 ; 12 (123)(567) (45)(67)	12 47 56 13 46 57 14 25 37 15 24 36 16 27 35 17 26 34 23 45 67
$\mathfrak{N}_{50}$; 5 ; 0 ; 3 (135)(267)	12 35 47 13 46 57 14 27 56 15 24 36 16 25 37 17 26 34 23 45 67	$\mathfrak{N}_{60}$; 6 ; 0 ; 6 (165437)	12 37 46 13 24 56 14 36 57 15 27 34 16 25 47 17 26 35 23 45 67
$\mathfrak{N}_{62}$; 6 ; 2 ; 42 (01)(45)(67) (153)(476)	01 45 67 03 16 47 04 17 35 05 37 46 06 15 34 07 13 56 14 36 57		

Table 4: The seven non–isomorphic near–one–factorizations of K_7; all of these result from contractions of the 11 overlarge sets of 2–(7,3,1) designs and from the derivation of the six one–factorizations of K_8. For the notation, see the beginning of Section 3.

5. Using extensions and contractions to define equivalence

For some of these systems, notably the OS(2–(9,3,1)), the number of isomorphism classes is large enough to be somewhat awkward. One obvious way to try for a coarser and more convenient equivalence relation on this set is the following:

> Suppose that A and B are two $OS(t$–$(v,k,1))$. Then A and B are said to be *equivalent* if one can be converted into the other by a finite sequence of contractions and extensions.

This definition works well in the smallest case, as shown in Figure 2. The sets of $NOF(K_7)$, OS(2–(7,3,1)) and OS(3–(8,4,1)) fall into three equivalence classes each, corresponding to the three components of the graph. However for the next case, the

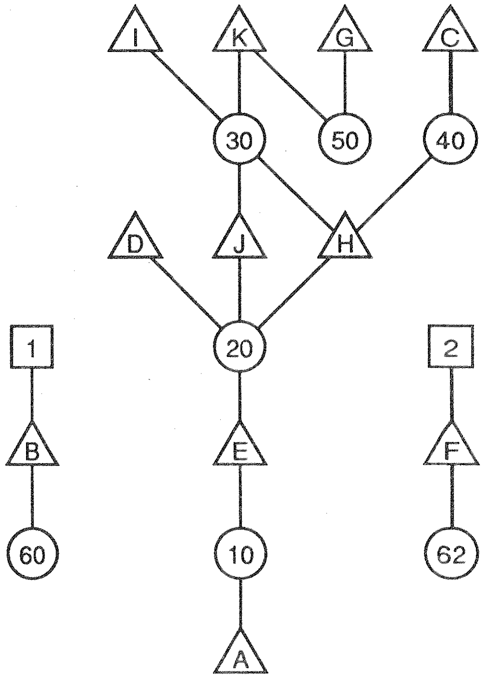

Figure 2: The relationships between near–one–factorizations of K_7 (denoted by circles), overlarge sets of 2–(7,3,1) designs (denoted by triangles), and overlarge sets of 3–(8,4,1) designs (denoted by squares).

extendible OS(2–(9,3,1)) all belong to the same component of the graph in Figure A1, and consequently are all equivalent to each other. (What is worse, they are all equivalent to the four non–extendible overlarge sets that also occur in the same component.) It follows from this that all the OS(3–(10,4,1)) are equivalent to each other, and so too are the OS(4–(11,5,1)).

Nevertheless, it is interesting to note the order in which the overlarge sets occur in the graphs. For instance, in Figures A2 and A3, the OS(3–(10,4,1)) appear in the following six layers, starting from that derived from a 4–(11,5,1) design :

21 ; 4 ; 3 ; 1, 2, 5, 6 ; 7, 8, 9, 10, 11, 16 ; 12, 13, 14, 15, 19, 20.

(Overlarge sets 17 and 18 appear in both cases in separate components.) In other words, given an OS(3–(10,4,1)), contracting it to an OS(2–(9,3,1)) and extending again has the same effect as extending it to an OS(4–(11,5,1)) and contracting again.

Similarly in Figure A2, the OS(2–(9,3,1)) appear in six layers, which are the same six layers as in the first part of Component 1 of Figure A1. The non–extendible overlarge sets $(18, 54, 58, 55)$ then link those of the two small components of Figure A2 into Component 1 of Figure A1.

6. Acknowledgements

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NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_1$; 1 ; 0 ; 48 (26359784) (34)(58)(67)	12 34 56 78 13 24 57 69 14 23 68 79 15 26 37 89 16 25 39 48 17 28 35 49 18 27 46 59 19 36 47 58 29 38 45 67	$\mathfrak{N}_2$; 1 ; 1 ; 432 (023)(489)(576) (08)(56)(79)	20 36 47 58 23 50 68 79 24 57 69 80 25 39 48 70 26 37 40 89 27 30 46 59 28 35 49 60 29 38 45 67 34 56 78 90
$\mathfrak{N}_3$; 2 ; 1 ; 12 (09)(35)(46) (34)(58)(67)	20 38 45 67 23 50 68 79 24 57 69 80 25 39 48 70 26 37 40 89 27 30 46 59 28 35 49 60 29 36 47 58 34 56 78 90	$\mathfrak{N}_4$; 2 ; 2 ; 12 (09)(35)(46) (34)(58)(67)	10 36 47 58 13 57 69 80 14 50 68 79 15 37 40 89 16 39 48 70 17 35 49 60 18 30 46 59 19 38 45 67 34 56 78 90
$\mathfrak{N}_5$; 4 ; 9 ; 2 (38)(47)(56)	10 35 47 68 12 34 56 78 13 24 57 80 14 23 58 60 15 26 37 40 16 25 48 70 17 28 36 50 18 27 30 46 20 38 45 67	$\mathfrak{N}_6$; 5 ; 0 ; 2 (36)(45)(78)	12 34 56 78 13 24 57 69 14 23 68 79 15 26 37 89 16 25 39 48 17 28 46 59 18 27 35 49 19 38 45 67 29 36 47 58
$\mathfrak{N}_7$; 5 ; 1 ; 8 (08)(56)(79) (36)(45)(78)	20 38 45 67 23 50 68 79 24 57 69 80 25 39 48 70 26 37 40 89 27 35 49 60 28 30 46 59 29 36 47 58 34 56 78 90	$\mathfrak{N}_8$; 6 ; 3 ; 2 (08)(56)(79)	10 29 47 68 12 56 78 90 14 58 60 79 15 26 40 89 16 25 48 70 17 20 46 59 18 27 49 50 19 28 45 67 24 57 69 80
$\mathfrak{N}_9$; 6 ; 7 ; 1 (1)	10 29 35 68 12 34 56 90 13 24 69 80 14 23 58 60 15 26 40 89 16 25 39 48 18 36 49 50 19 28 30 45 20 38 46 59	$\mathfrak{N}_{10}$; 8 ; 0 ; 2 (23)(67)(89)	12 34 56 78 13 24 57 69 14 23 68 79 15 26 37 89 16 27 49 58 17 36 48 59 18 25 39 47 19 28 35 46 29 38 45 67

Table A1: The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{11}$; 8 ; 1 ; 4 (05)(69)(78) (23)(67)(89)	20 36 48 59 23 50 68 79 24 57 69 80 25 39 47 60 26 37 40 89 27 30 49 58 28 35 46 70 29 38 45 67 34 56 78 90	$\mathfrak{N}_{12}$; 8 ; 4 ; 4 (05)(69)(78) (23)(67)(89)	10 29 38 67 12 56 78 90 13 57 69 80 15 26 37 89 16 27 30 58 17 20 36 59 18 25 39 60 19 28 35 70 23 50 68 79
$\mathfrak{N}_{13}$; 9 ; 1 ; 4 (06)(24)(89) (09)(57)(68)	20 38 45 67 23 50 68 79 24 57 69 80 25 39 46 70 26 37 40 89 27 30 48 59 28 35 49 60 29 36 47 58 34 56 78 90	$\mathfrak{N}_{14}$; 9 ; 3 ; 4 (06)(24)(89) (09)(57)(68)	10 29 47 58 12 56 78 90 14 50 68 79 15 26 40 89 16 27 48 59 17 28 49 60 18 25 46 70 19 20 45 67 24 57 69 80
$\mathfrak{N}_{15}$; 10 ; 2 ; 2 (08)(56)(79)	10 36 47 89 13 57 69 80 14 58 60 79 15 39 40 68 16 37 48 50 17 30 46 59 18 35 49 70 19 38 45 67 34 56 78 90	$\mathfrak{N}_{16}$; 10 ; 3 ; 2 (08)(56)(79)	10 25 47 89 12 56 78 90 14 58 60 79 15 27 40 68 16 29 48 50 17 28 46 59 18 26 49 70 19 20 45 67 24 57 69 80
$\mathfrak{N}_{17}$; 10 ; 7 ; 1 (1)	10 25 36 89 12 34 56 90 13 24 69 80 14 23 58 60 15 39 40 68 16 29 48 50 18 26 35 49 19 20 38 45 28 30 46 59	$\mathfrak{N}_{18}$; 11 ; 1 ; 6 (08)(56)(79) (09)(57)(68)	20 39 45 67 23 50 68 79 24 57 69 80 25 36 40 89 26 35 48 70 27 38 49 60 28 37 46 59 29 30 47 58 34 56 78 90
$\mathfrak{N}_{19}$; 12 ; 1 ; 4 (08)(56)(79) (09)(34)(56)(78)	20 35 49 67 23 50 68 79 24 57 69 80 25 37 40 89 26 39 48 70 27 38 45 60 28 36 47 59 29 30 46 58 34 56 78 90	$\mathfrak{N}_{20}$; 12 ; 2 ; 12 (03)(16)(47) (09)(34)(56)(78)	10 36 47 59 13 57 69 80 14 50 68 79 15 39 48 70 16 37 40 89 17 30 46 58 18 35 49 67 19 38 45 60 34 56 78 90

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{21}$; 13 ; 3 ; 6 (086579)(124)	10 29 47 58 12 56 78 90 14 50 68 79 15 26 48 70 16 27 45 89 17 25 49 60 18 20 46 59 19 28 40 67 24 57 69 80	$\mathfrak{N}_{22}$; 16 ; 8 ; 1 (1)	10 29 35 47 12 34 56 90 13 24 57 69 14 23 60 79 15 26 37 40 16 27 39 50 17 20 46 59 19 36 45 70 25 30 49 67
$\mathfrak{N}_{23}$; 18 ; 1 ; 2 (05)(69)(78)	20 36 47 59 23 50 68 79 24 57 69 80 25 39 48 60 26 37 40 89 27 30 49 58 28 35 46 70 29 38 45 67 34 56 78 90	$\mathfrak{N}_{24}$; 18 ; 4 ; 2 (05)(69)(78)	10 29 38 67 12 56 78 90 13 57 69 80 15 26 37 89 16 27 30 58 17 25 39 60 18 20 36 59 19 28 35 70 23 50 68 79
$\mathfrak{N}_{25}$; 19 ; 2 ; 2 (08)(56)(79)	10 36 49 58 13 57 69 80 14 50 68 79 15 37 40 89 16 39 48 70 17 30 46 59 18 35 47 60 19 38 45 67 34 56 78 90	$\mathfrak{N}_{26}$; 19 ; 3 ; 2 (08)(56)(79)	10 27 49 58 12 56 78 90 14 50 68 79 15 26 40 89 16 25 48 70 17 28 46 59 18 29 47 60 19 20 45 67 24 57 69 80
$\mathfrak{N}_{27}$; 19 ; 4 ; 2 (08)(56)(79)	10 27 36 58 12 56 78 90 13 57 69 80 15 26 37 89 16 25 39 70 17 28 30 59 18 29 35 60 19 20 38 67 23 50 68 79	$\mathfrak{N}_{28}$; 21 ; 1 ; 2 (08)(56)(79)	20 36 47 59 23 58 60 79 24 57 69 80 25 39 48 70 26 37 40 89 27 38 46 50 28 35 49 67 29 30 45 68 34 56 78 90
$\mathfrak{N}_{29}$; 21 ; 2 ; 2 (08)(56)(79)	10 35 49 67 13 57 69 80 14 58 60 79 15 37 40 89 16 39 48 70 17 30 45 68 18 36 47 59 19 38 46 50 34 56 78 90	$\mathfrak{N}_{30}$; 22 ; 0 ; 8 (16482537)	12 34 56 78 13 25 47 69 14 26 37 89 15 23 48 79 16 24 38 59 17 29 45 68 18 35 49 67 19 28 36 57 27 39 46 58

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{31}$; 22 ; 9 ; 8 (15472638)	10 27 46 58 12 34 56 78 13 25 47 80 14 26 37 50 15 23 48 60 16 24 38 70 17 30 45 68 18 20 35 67 28 36 40 57	$\mathfrak{N}_{32}$; 23 ; 3 ; 2 (08)(56)(79)	10 27 49 68 12 56 78 90 14 58 60 79 15 26 40 89 16 25 48 70 17 20 46 59 18 29 47 50 19 28 45 67 24 57 69 80
$\mathfrak{N}_{33}$; 24 ; 1 ; 2 (08)(56)(79)	20 38 45 67 23 58 60 79 24 57 69 80 25 39 47 68 26 37 49 50 27 36 40 89 28 30 46 59 29 35 48 70 34 56 78 90	$\mathfrak{N}_{34}$; 24 ; 3 ; 2 (08)(56)(79)	10 25 47 68 12 56 78 90 14 58 60 79 15 27 40 89 16 29 48 70 17 28 46 59 18 26 49 50 19 20 45 67 24 57 69 80
$\mathfrak{N}_{35}$; 24 ; 4 ; 2 (08)(56)(79)	10 25 39 68 12 56 78 90 13 57 69 80 15 27 36 89 16 29 35 70 17 28 30 59 18 26 37 50 19 20 38 67 23 58 60 79	$\mathfrak{N}_{36}$; 27 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 23 58 79 15 26 37 89 16 29 38 47 17 28 46 59 18 25 36 49 19 35 48 67 27 39 45 68
$\mathfrak{N}_{37}$; 34 ; 2 ; 1 (1)	10 36 48 59 13 57 69 80 14 50 68 79 15 37 40 89 16 35 49 70 17 30 46 58 18 39 45 67 19 38 47 60 34 56 78 90	$\mathfrak{N}_{38}$; 37 ; 2 ; 1 (1)	10 38 49 67 13 57 69 80 14 50 68 79 15 37 40 89 16 30 48 59 17 39 46 58 18 36 45 70 19 35 47 60 34 56 78 90
$\mathfrak{N}_{39}$; 40 ; 1 ; 1 (1)	20 38 45 67 23 50 68 79 24 57 69 80 25 39 48 60 26 37 40 89 27 30 49 58 28 36 47 59 29 35 46 70 34 56 78 90	$\mathfrak{N}_{40}$; 40 ; 2 ; 1 (1)	10 36 47 59 13 57 69 80 14 50 68 79 15 37 40 89 16 30 49 58 17 39 48 60 18 35 46 70 19 38 45 67 34 56 78 90

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{41}$; 43 ; 9 ; 1 (1)	10 27 38 46 12 34 56 78 13 25 47 80 14 26 37 50 15 23 48 60 16 24 58 70 17 30 45 68 18 20 35 67 28 36 40 57	$\mathfrak{N}_{42}$; 44 ; 1 ; 2 (34)(58)(67)	20 38 45 67 23 50 68 79 24 57 69 80 25 39 48 60 26 37 40 89 27 30 46 59 28 35 49 70 29 36 47 58 34 56 78 90
$\mathfrak{N}_{43}$; 48 ; 3 ; 1 (1)	10 25 47 68 12 56 78 90 14 58 60 79 15 26 40 89 16 29 48 70 17 28 46 59 18 27 49 50 19 20 45 67 24 57 69 80	$\mathfrak{N}_{44}$; 49 ; 0 ; 2 (12)(34)(56)(78)	12 34 56 78 13 25 47 69 14 26 37 89 15 23 48 79 16 24 38 59 17 29 35 68 18 39 45 67 19 28 46 57 27 36 49 58
$\mathfrak{N}_{45}$; 50 ; 3 ; 1 (1)	10 26 47 59 12 56 78 90 14 25 70 89 15 48 60 79 16 27 49 58 17 29 40 68 18 20 45 67 19 28 46 50 24 57 69 80	$\mathfrak{N}_{46}$; 51 ; 3 ; 2 (08)(56)(79)	10 25 49 68 12 56 78 90 14 58 60 79 15 27 40 89 16 29 48 70 17 28 46 59 18 26 47 50 19 20 45 67 24 57 69 80
$\mathfrak{N}_{47}$; 51 ; 4 ; 2 (08)(56)(79)	10 25 37 68 12 56 78 90 13 57 69 80 15 27 36 89 16 29 35 70 17 28 30 59 18 26 39 50 19 20 38 67 23 58 60 79	$\mathfrak{N}_{48}$; 53 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 25 37 89 15 26 38 79 16 27 48 59 17 28 39 46 18 35 49 67 19 23 45 68 29 36 47 58
$\mathfrak{N}_{49}$; 53 ; 5 ; 1 (1)	10 29 36 47 12 34 78 90 13 24 69 80 14 37 60 89 16 27 30 48 17 28 39 46 18 20 49 67 19 23 68 70 26 38 40 79	$\mathfrak{N}_{50}$; 55 ; 2 ; 1 (1)	10 39 45 67 13 57 69 80 14 58 60 79 15 30 49 68 16 38 47 59 17 35 40 89 18 37 46 50 19 36 48 70 34 56 78 90

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{51}$; 59 ; 9 ; 1 (1)	10 27 45 68 12 34 56 78 13 24 57 80 14 25 36 70 15 23 48 60 16 38 47 50 17 26 30 58 18 20 37 46 28 35 40 67	$\mathfrak{N}_{52}$; 60 ; 2 ; 1 (1)	10 38 45 67 13 57 69 80 14 36 70 89 15 40 68 79 16 39 48 50 17 35 49 60 18 37 46 59 19 30 47 58 34 56 78 90
$\mathfrak{N}_{53}$; 63 ; 8 ; 1 (1)	10 25 39 47 12 34 56 90 13 24 57 69 14 23 60 79 15 26 37 40 16 27 49 50 17 30 46 59 19 20 35 67 29 36 45 70	$\mathfrak{N}_{54}$; 64 ; 8 ; 1 (1)	10 27 36 59 12 34 56 90 13 24 57 69 14 23 60 79 15 26 37 40 16 39 47 50 17 25 30 49 19 20 45 67 29 35 46 70
$\mathfrak{N}_{55}$; 65 ; 0 ; 1 (1)	12 34 56 78 13 25 47 69 14 26 37 89 15 23 68 79 16 28 39 45 17 36 49 58 18 29 46 57 19 27 35 48 24 38 59 67	$\mathfrak{N}_{56}$; 67 ; 3 ; 1 (1)	10 29 48 67 12 56 78 90 14 58 60 79 15 26 40 89 16 20 47 59 17 28 49 50 18 25 46 70 19 27 45 68 24 57 69 80
$\mathfrak{N}_{57}$; 73 ; 3 ; 1 (1)	10 29 48 67 12 56 78 90 14 50 68 79 15 26 40 89 16 28 47 59 17 20 46 58 18 25 49 70 19 27 45 60 24 57 69 80	$\mathfrak{N}_{58}$; 78 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 25 36 89 15 23 68 79 16 27 49 58 17 29 35 48 18 39 45 67 19 26 38 47 28 37 46 59
$\mathfrak{N}_{59}$; 84 ; 4 ; 1 (1)	10 28 36 59 12 56 78 90 13 57 69 80 15 26 39 70 16 27 35 89 17 25 38 60 18 29 30 67 19 20 37 58 23 50 68 79	$\mathfrak{N}_{60}$; 85 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 25 36 89 15 23 68 79 16 27 49 58 17 29 35 48 18 37 46 59 19 26 38 47 28 39 45 67

Table A1(cont'd): The 150 non-isomorphic near-one-factorizations of K_9 which result from contractions of the 77 overlarge sets of 2-(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{61}$; 91 ; 3 ; 1 (1)	10 27 46 59 12 56 78 90 14 26 50 89 15 40 68 79 16 24 58 70 17 29 48 60 18 20 49 57 19 28 45 67 25 47 69 80	$\mathfrak{N}_{62}$; 91 ; 5 ; 1 (1)	10 27 38 46 12 34 78 90 13 47 69 80 14 26 37 89 16 24 39 70 17 29 48 60 18 20 36 49 19 28 30 67 23 40 68 79
$\mathfrak{N}_{63}$; 91 ; 9 ; 1 (1)	10 27 38 46 12 34 56 78 13 25 47 80 14 26 37 50 15 23 40 68 16 24 58 70 17 35 48 60 18 20 36 57 28 30 45 67	$\mathfrak{N}_{64}$; 92 ; 1 ; 1 (1)	20 37 48 59 23 49 58 60 24 57 69 80 25 36 70 89 26 38 40 79 27 39 46 50 28 30 45 67 29 35 47 68 34 56 78 90
$\mathfrak{N}_{65}$; 95 ; 1 ; 2 (08)(56)(79)	20 37 45 68 23 58 60 79 24 57 69 80 25 30 49 67 26 38 47 59 27 36 40 89 28 39 46 50 29 35 48 70 34 56 78 90	$\mathfrak{N}_{66}$; 95 ; 3 ; 2 (08)(56)(79)	10 26 47 59 12 56 78 90 14 58 60 79 15 27 40 89 16 29 48 70 17 28 46 50 18 25 49 67 19 20 45 68 24 57 69 80
$\mathfrak{N}_{67}$; 96 ; 3 ; 1 (1)	10 29 48 67 12 56 78 90 14 58 60 79 15 26 40 89 16 20 47 59 17 28 46 50 18 25 49 70 19 27 45 68 24 57 69 80	$\mathfrak{N}_{68}$; 97 ; 5 ; 1 (1)	10 29 47 68 12 34 78 90 13 24 69 80 14 36 70 89 16 27 30 49 17 28 39 60 18 20 37 46 19 23 48 67 26 38 40 79
$\mathfrak{N}_{69}$; 98 ; 3 ; 2 (08)(56)(79)	10 27 49 68 12 56 78 90 14 58 60 79 15 26 40 89 16 25 48 70 17 28 46 59 18 29 47 50 19 20 45 67 24 57 69 80	$\mathfrak{N}_{70}$; 99 ; 4 ; 1 (1)	10 29 35 67 12 56 78 90 13 57 69 80 15 26 37 89 16 27 30 58 17 28 39 60 18 20 36 59 19 25 38 70 23 50 68 79

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{71}$; 104 ; 8 ; 1 (1)	10 27 36 59 12 34 56 90 13 25 47 69 14 26 39 70 15 30 46 79 16 20 37 45 17 29 35 40 19 24 57 60 23 49 50 67	$\mathfrak{N}_{72}$; 105 ; 3 ; 1 (1)	10 28 49 67 12 56 78 90 14 25 70 89 15 40 68 79 16 20 48 59 17 29 46 50 18 27 45 60 19 26 47 58 24 57 69 80
$\mathfrak{N}_{73}$; 106 ; 2 ; 1 (1)	10 38 49 67 13 57 69 80 14 58 60 79 15 37 40 89 16 35 48 70 17 39 46 50 18 36 47 59 19 30 45 68 34 56 78 90	$\mathfrak{N}_{74}$; 114 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 25 36 89 15 26 48 79 16 29 37 58 17 28 39 45 18 35 49 67 19 27 38 46 23 47 59 68
$\mathfrak{N}_{75}$; 117 ; 5 ; 1 (1)	10 28 37 46 12 34 78 90 13 24 69 80 14 23 60 79 16 29 38 47 17 26 30 89 18 20 49 67 19 36 48 70 27 39 40 68	$\mathfrak{N}_{76}$; 121 ; 6 ; 1 (1)	10 28 49 57 12 34 78 90 13 25 47 80 14 37 50 89 15 23 48 79 17 20 38 59 18 29 30 45 19 27 35 40 24 39 58 70
$\mathfrak{N}_{77}$; 125 ; 8 ; 1 (1)	10 27 39 45 12 34 56 90 13 24 57 69 14 25 36 70 15 26 40 79 16 30 47 59 17 29 35 60 19 20 37 46 23 49 50 67	$\mathfrak{N}_{78}$; 129 ; 3 ; 1 (1)	10 26 48 59 12 56 78 90 14 25 70 89 15 40 68 79 16 29 47 58 17 28 45 60 18 20 49 67 19 27 46 50 24 57 69 80
$\mathfrak{N}_{79}$; 130 ; 3 ; 1 (1)	10 25 47 68 12 56 78 90 14 58 60 79 15 26 40 89 16 29 48 70 17 20 46 59 18 27 49 50 19 28 45 67 24 57 69 80	$\mathfrak{N}_{80}$; 131 ; 7 ; 1 (1)	10 23 45 68 12 34 56 90 13 24 69 80 14 25 36 89 15 38 49 60 16 28 30 59 18 29 46 50 19 20 35 48 26 39 40 58

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{81}$; 132 ; 1 ; 2 (08)(56)(79)	20 39 45 67 23 58 60 79 24 57 69 80 25 36 48 70 26 35 40 89 27 30 49 68 28 37 46 59 29 38 47 50 34 56 78 90	$\mathfrak{N}_{82}$; 132 ; 2 ; 6 (076895)(143)	10 37 46 59 13 57 69 80 14 58 60 79 15 30 49 68 16 38 47 50 17 35 40 89 18 39 45 67 19 36 48 70 34 56 78 90
$\mathfrak{N}_{83}$; 135 ; 1 ; 2 (08)(56)(79)	20 35 49 67 23 50 68 79 24 57 69 80 25 37 40 89 26 39 48 70 27 30 46 58 28 36 47 59 29 38 45 60 34 56 78 90	$\mathfrak{N}_{84}$; 135 ; 2 ; 2 (08)(56)(79)	10 36 47 59 13 57 69 80 14 50 68 79 15 39 48 70 16 37 40 89 17 38 45 60 18 35 49 67 19 30 46 58 34 56 78 90
$\mathfrak{N}_{85}$; 136 ; 1 ; 2 (05)(69)(78)	20 37 46 59 23 50 68 79 24 57 69 80 25 38 49 60 26 39 48 70 27 35 40 89 28 30 45 67 29 36 47 58 34 56 78 90	$\mathfrak{N}_{86}$; 139 ; 2 ; 1 (1)	10 39 47 58 13 57 69 80 14 36 70 89 15 40 68 79 16 37 49 50 17 35 48 60 18 30 46 59 19 38 45 67 34 56 78 90
$\mathfrak{N}_{87}$; 139 ; 8 ; 1 (1)	10 26 39 47 12 34 56 90 13 24 57 69 14 25 36 70 15 23 40 79 16 37 49 50 17 29 35 60 19 20 45 67 27 30 46 59	$\mathfrak{N}_{88}$; 140 ; 3 ; 1 (1)	10 25 49 68 12 56 78 90 14 58 60 79 15 27 40 89 16 29 48 70 17 28 46 50 18 26 47 59 19 20 45 67 24 57 69 80
$\mathfrak{N}_{89}$; 142 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 25 36 89 15 23 68 79 16 29 38 47 17 26 49 58 18 39 45 67 19 27 35 48 28 37 46 59	$\mathfrak{N}_{90}$; 145 ; 0 ; 1 (1)	12 34 56 78 13 24 57 69 14 25 36 89 15 23 68 79 16 29 38 47 17 26 49 58 18 37 46 59 19 27 35 48 28 39 45 67

Table A1(cont'd): The 150 non-isomorphic near-one-factorizations of K_9 which result from contractions of the 77 overlarge sets of 2-(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{91}$; 146 ; 4 ; 1 (1)	10 27 38 59 12 56 78 90 13 57 69 80 15 26 30 79 16 29 37 58 17 28 35 60 18 20 39 67 19 23 50 68 25 36 70 89	$\mathfrak{N}_{92}$; 147 ; 1 ; 1 (1)	20 35 47 68 23 48 59 70 24 57 69 80 25 37 60 89 26 38 40 79 27 36 49 58 28 39 46 50 29 30 45 67 34 56 78 90
$\mathfrak{N}_{93}$; 148 ; 1 ; 2 (05)(69)(78)	20 38 46 59 23 50 68 79 24 57 69 80 25 37 49 60 26 39 48 70 27 35 40 89 28 30 45 67 29 36 47 58 34 56 78 90	$\mathfrak{N}_{94}$; 148 ; 2 ; 2 (05)(69)(78)	10 36 47 58 13 57 69 80 14 50 68 79 15 39 48 70 16 35 40 89 17 38 46 59 18 37 49 60 19 30 45 67 34 56 78 90
$\mathfrak{N}_{95}$; 149 ; 8 ; 1 (1)	10 29 45 67 12 34 56 90 13 24 57 69 14 25 36 70 15 23 40 79 16 20 39 47 17 26 30 59 19 37 46 50 27 35 49 60	$\mathfrak{N}_{96}$; 150 ; 1 ; 2 (09)(34)(56)(78)	20 37 46 59 23 49 68 70 24 30 57 89 25 47 69 80 26 38 50 79 27 36 40 58 28 39 45 67 29 35 48 60 34 56 78 90
$\mathfrak{N}_{97}$; 151 ; 5 ; 1 (1)	10 28 39 67 12 34 78 90 13 24 69 80 14 36 70 89 16 27 30 49 17 23 48 60 18 29 37 46 19 20 47 68 26 38 40 79	$\mathfrak{N}_{98}$; 174 ; 9 ; 1 (1)	10 26 37 48 12 34 56 78 13 24 57 80 14 25 36 70 15 23 40 68 16 20 47 58 17 38 46 50 18 27 35 60 28 30 45 67
$\mathfrak{N}_{99}$; 177 ; 6 ; 1 (1)	10 29 47 58 12 34 78 90 13 24 57 80 14 25 37 89 15 38 40 79 17 28 39 50 18 20 35 49 19 27 30 45 23 48 59 70	$\mathfrak{N}_{100}$; 180 ; 7 ; 1 (1)	10 28 46 59 12 34 56 90 13 24 69 80 14 25 36 89 15 23 40 68 16 30 49 58 18 26 39 50 19 20 38 45 29 35 48 60

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{101}$; 199 ; 0 ; 1 (1)	12 34 56 78 13 25 47 69 14 26 38 79 15 27 36 89 16 29 48 57 17 24 39 58 18 37 46 59 19 23 45 68 28 35 49 67	$\mathfrak{N}_{102}$; 205 ; 2 ; 1 (1)	10 38 46 59 13 47 69 80 14 37 50 89 15 39 68 70 16 35 40 79 17 36 49 58 18 30 45 67 19 48 57 60 34 56 78 90
$\mathfrak{N}_{103}$; 206 ; 2 ; 1 (1)	10 39 45 67 13 47 69 80 14 30 58 79 15 36 40 89 16 35 48 70 17 49 50 68 18 37 46 59 19 38 57 60 34 56 78 90	$\mathfrak{N}_{104}$; 206 ; 4 ; 1 (1)	10 28 39 67 12 56 78 90 13 25 69 80 15 27 36 89 16 29 35 70 17 23 50 68 18 20 37 59 19 38 57 60 26 30 58 79
$\mathfrak{N}_{105}$; 209 ; 5 ; 1 (1)	10 27 39 46 12 34 78 90 13 47 69 80 14 26 37 89 16 28 40 79 17 20 36 48 18 29 30 67 19 24 38 60 23 49 68 70	$\mathfrak{N}_{106}$; 214 ; 1 ; 2 (09)(34)(56)(78)	20 35 49 68 23 48 59 67 24 37 58 60 25 47 69 80 26 38 50 79 27 36 40 89 28 39 45 70 29 30 46 57 34 56 78 90
$\mathfrak{N}_{107}$; 215 ; 7 ; 1 (1)	10 29 35 48 12 34 56 90 13 24 69 80 14 25 36 89 15 23 40 68 16 20 39 58 18 30 46 59 19 28 45 60 26 38 49 50	$\mathfrak{N}_{108}$; 217 ; 5 ; 1 (1)	10 28 37 46 12 34 78 90 13 24 69 80 14 36 70 89 16 27 30 49 17 20 39 68 18 29 47 60 19 23 48 67 26 38 40 79
$\mathfrak{N}_{109}$; 217 ; 6 ; 1 (1)	10 28 37 59 12 34 78 90 13 24 57 80 14 25 70 89 15 38 40 79 17 20 39 45 18 29 35 47 19 23 48 50 27 30 49 58	$\mathfrak{N}_{110}$; 218 ; 8 ; 1 (1)	10 27 36 49 12 34 56 90 13 25 47 69 14 26 37 50 15 24 30 79 16 39 45 70 17 20 46 59 19 23 57 60 29 35 40 67

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{111}$; 221 ; 1 ; 1 (1)	20 37 49 58 23 48 60 79 24 57 69 80 25 36 70 89 26 38 47 50 27 39 45 68 28 30 46 59 29 35 40 67 34 56 78 90	$\mathfrak{N}_{112}$; 225 ; 2 ; 1 (1)	10 37 46 59 13 47 69 80 14 38 50 79 15 39 68 70 16 35 40 89 17 36 49 58 18 30 45 67 19 48 57 60 34 56 78 90
$\mathfrak{N}_{113}$; 229 ; 2 ; 1 (1)	10 36 47 59 13 57 69 80 14 50 68 79 15 39 48 70 16 35 40 89 17 30 46 58 18 37 49 60 19 38 45 67 34 56 78 90	$\mathfrak{N}_{114}$; 231 ; 2 ; 1 (1)	10 39 46 58 13 57 69 80 14 36 70 89 15 38 40 79 16 37 48 59 17 49 50 68 18 35 47 60 19 30 45 67 34 56 78 90
$\mathfrak{N}_{115}$; 234 ; 1 ; 4 (08)(56)(79) (09)(34)(56)(78)	20 36 47 59 23 50 68 79 24 57 69 80 25 37 40 89 26 39 48 70 27 30 46 58 28 35 49 67 29 38 45 60 34 56 78 90	$\mathfrak{N}_{116}$; 234 ; 2 ; 4 (08)(56)(79) (09)(34)(56)(78)	10 35 49 67 13 57 69 80 14 50 68 79 15 39 48 70 16 37 40 89 17 38 45 60 18 36 47 59 19 30 46 58 34 56 78 90
$\mathfrak{N}_{117}$; 236 ; 3 ; 1 (1)	10 26 48 79 12 56 78 90 14 25 60 89 15 28 46 70 16 27 40 59 17 29 45 68 18 49 50 67 19 20 47 58 24 57 69 80	$\mathfrak{N}_{118}$; 250 ; 0 ; 1 (1)	12 34 56 78 13 25 47 69 14 26 37 89 15 23 49 68 16 35 48 79 17 29 36 58 18 24 39 57 19 28 45 67 27 38 46 59
$\mathfrak{N}_{119}$; 251 ; 0 ; 1 (1)	12 34 56 78 13 25 47 69 14 26 38 79 15 23 49 68 16 27 48 59 17 39 46 58 18 29 37 45 19 28 35 67 24 36 57 89	$\mathfrak{N}_{120}$; 252 ; 4 ; 1 (1)	10 28 39 67 12 56 78 90 13 25 69 80 15 23 68 79 16 20 38 57 17 29 30 58 18 36 59 70 19 27 35 60 26 37 50 89

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{121}$; 257 ; 4 ; 1 (1)	10 27 36 59 12 56 78 90 13 25 69 80 15 23 68 70 16 20 37 89 17 29 30 58 18 39 57 60 19 28 35 67 26 38 50 79	$\mathfrak{N}_{122}$; 268 ; 6 ; 1 (1)	10 28 35 49 12 34 78 90 13 25 47 80 14 37 50 89 15 23 40 79 17 29 38 45 18 24 59 70 19 27 30 58 20 39 48 57
$\mathfrak{N}_{123}$; 282 ; 1 ; 1 (1)	20 39 46 58 23 40 68 79 24 57 69 80 25 37 60 89 26 35 49 70 27 36 48 59 28 30 45 67 29 38 47 50 34 56 78 90	$\mathfrak{N}_{124}$; 283 ; 5 ; 1 (1)	10 26 37 89 12 34 78 90 13 24 69 80 14 38 60 79 16 29 30 47 17 20 36 48 18 27 39 46 19 28 40 67 23 49 68 70
$\mathfrak{N}_{125}$; 286 ; 7 ; 1 (1)	10 26 39 58 12 34 56 90 13 24 69 80 14 25 36 89 15 38 49 60 16 23 48 50 18 20 46 59 19 28 35 40 29 30 45 68	$\mathfrak{N}_{126}$; 293 ; 7 ; 1 (1)	10 28 35 49 12 34 56 90 13 25 69 80 14 26 50 89 15 23 40 68 16 20 39 48 18 24 36 59 19 38 45 60 29 30 46 58
$\mathfrak{N}_{127}$; 294 ; 2 ; 1 (1)	10 35 68 79 13 47 69 80 14 39 58 70 15 30 48 67 16 38 40 59 17 46 50 89 18 36 49 57 19 37 45 60 34 56 78 90	$\mathfrak{N}_{128}$; 297 ; 6 ; 1 (1)	10 23 48 59 12 34 78 90 13 25 47 80 14 37 50 89 15 24 39 70 17 20 49 58 18 29 30 57 19 27 38 45 28 35 40 79
$\mathfrak{N}_{129}$; 311 ; 8 ; 1 (1)	10 27 36 59 12 34 56 90 13 25 47 69 14 26 39 70 15 30 46 79 16 20 37 45 17 23 49 50 19 24 57 60 29 35 40 67	$\mathfrak{N}_{130}$; 313 ; 2 ; 1 (1)	10 37 46 59 13 47 69 80 14 30 58 79 15 36 40 89 16 35 48 70 17 49 50 68 18 39 45 67 19 38 57 60 34 56 78 90

Table A1(cont'd): The 150 non–isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{131}$; 319 ; 6 ; 1 (1)	10 27 48 59 12 34 78 90 13 25 47 80 14 39 58 70 15 28 30 79 17 29 35 40 18 23 49 57 19 24 38 50 20 37 45 89	$\mathfrak{N}_{132}$; 328 ; 0 ; 4 (1324)(5867)	12 34 56 78 13 25 47 69 14 26 37 89 15 23 48 79 16 24 38 59 17 29 36 58 18 39 46 57 19 28 45 67 27 35 49 68
$\mathfrak{N}_{133}$; 329 ; 1 ; 4 (0394)(5768)	20 39 45 67 23 49 68 70 24 30 57 89 25 47 69 80 26 38 50 79 27 35 48 60 28 37 46 59 29 36 40 58 34 56 78 90	$\mathfrak{N}_{134}$; 329 ; 2 ; 4 (0394)(5768)	10 37 46 59 13 47 69 80 14 38 50 79 15 49 68 70 16 30 57 89 17 36 40 58 18 39 45 67 19 35 48 60 34 56 78 90
$\mathfrak{N}_{135}$; 330 ; 0 ; 1 (1)	12 34 56 78 13 25 47 69 14 26 38 79 15 24 39 68 16 23 57 89 17 36 49 58 18 29 45 67 19 27 35 48 28 37 46 59	$\mathfrak{N}_{136}$; 330 ; 1 ; 4 (0394)(5768)	20 36 49 58 23 40 57 89 24 39 68 70 25 47 69 80 26 38 50 79 27 35 48 60 28 37 46 59 29 30 45 67 34 56 78 90
$\mathfrak{N}_{137}$; 338 ; 6 ; 1 (1)	10 23 58 79 12 34 78 90 13 25 47 80 14 37 50 89 15 27 38 49 17 20 39 45 18 24 59 70 19 28 35 40 29 30 48 57	$\mathfrak{N}_{138}$; 348 ; 7 ; 1 (1)	10 26 48 59 12 34 56 90 13 24 69 80 14 25 38 60 15 23 49 68 16 35 40 89 18 29 36 50 19 28 30 45 20 39 46 58
$\mathfrak{N}_{139}$; 349 ; 0 ; 1 (1)	12 34 56 78 13 25 47 69 14 26 37 89 15 24 39 68 16 28 35 79 17 38 46 59 18 29 45 67 19 23 48 57 27 36 49 58	$\mathfrak{N}_{140}$; 350 ; 9 ; 1 (1)	10 28 37 46 12 34 56 78 13 25 47 80 14 26 38 50 15 23 68 70 16 30 48 57 17 24 35 60 18 20 45 67 27 36 40 58

Table A1(cont'd): The 150 non-isomorphic near–one–factorizations of K_9 which result from contractions of the 77 overlarge sets of 2–(9,3,1) designs. For the notation see the beginning of Section 4.

NOF ; index ; i ; $\mid G \mid$; generators	Blocks	NOF ; index ; i ; $\mid G \mid$; generators	Blocks
$\mathfrak{N}_{141}$; 356 ; 6 ; 1 (1)	10 24 57 89 12 34 78 90 13 25 47 80 14 39 58 70 15 29 38 40 17 20 35 49 18 23 45 79 19 28 37 50 27 30 48 59	$\mathfrak{N}_{142}$; 359 ; 7 ; 1 (1)	10 24 38 59 12 34 56 90 13 25 69 80 14 26 39 58 15 36 40 89 16 20 35 48 18 23 49 60 19 28 46 50 29 30 45 68
$\mathfrak{N}_{143}$; 359 ; 9 ; 1 (1)	10 24 38 67 12 34 56 78 13 25 47 80 14 26 58 70 15 27 36 40 16 20 35 48 17 30 45 68 18 23 57 60 28 37 46 50	$\mathfrak{N}_{144}$; 362 ; 9 ; 3 (063)(175)(248)	10 28 37 46 12 34 56 78 13 25 47 80 14 26 58 70 15 27 36 40 16 20 35 48 17 30 45 68 18 23 57 60 24 38 50 67
$\mathfrak{N}_{145}$; 364 ; 6 ; 1 (1)	10 27 48 59 12 34 78 90 13 25 47 80 14 37 50 89 15 24 30 79 17 20 39 58 18 23 49 57 19 28 35 40 29 38 45 70	$\mathfrak{N}_{146}$; 380 ; 3 ; 6 (076895)(124)	10 25 49 68 12 56 78 90 14 58 60 79 15 27 40 89 16 29 48 70 17 20 46 59 18 26 47 50 19 28 45 67 24 57 69 80
$\mathfrak{N}_{147}$; 381 ; 4 ; 3 (078)(195)(263)	10 28 39 67 12 56 78 90 13 57 69 80 15 27 36 89 16 20 37 58 17 26 30 59 18 29 35 70 19 23 50 68 25 38 60 79	$\mathfrak{N}_{148}$; 386 ; 1 ; 1 (1)	20 39 45 67 23 49 68 70 24 36 58 79 25 47 69 80 26 37 50 89 27 38 40 59 28 30 46 57 29 35 48 60 34 56 78 90
$\mathfrak{N}_{149}$; 395 ; 1 ; 2 (09)(34)(56)(78)	20 35 49 67 23 40 57 89 24 39 68 70 25 47 69 80 26 38 50 79 27 36 48 59 28 37 45 60 29 30 46 58 34 56 78 90	$\mathfrak{N}_{150}$; 396 ; 0 ; 4 (1734)(2956)	12 34 56 78 13 25 47 69 14 26 38 79 15 24 39 68 16 23 57 89 17 29 35 48 18 37 46 59 19 28 45 67 27 36 49 58

Table A1(cont'd): The 150 non-isomorphic near-one-factorizations of K_9 which result from contractions of the 77 overlarge sets of 2-(9,3,1) designs. For the notation see the beginning of Section 4.

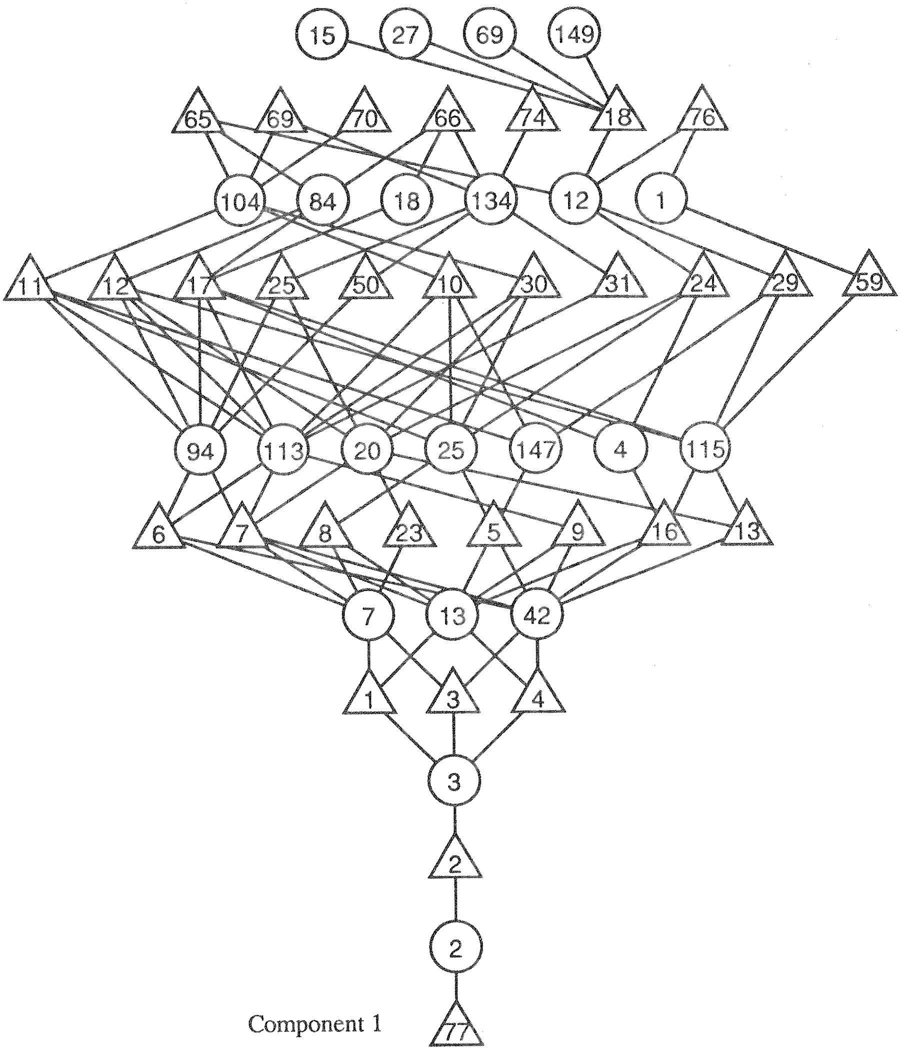
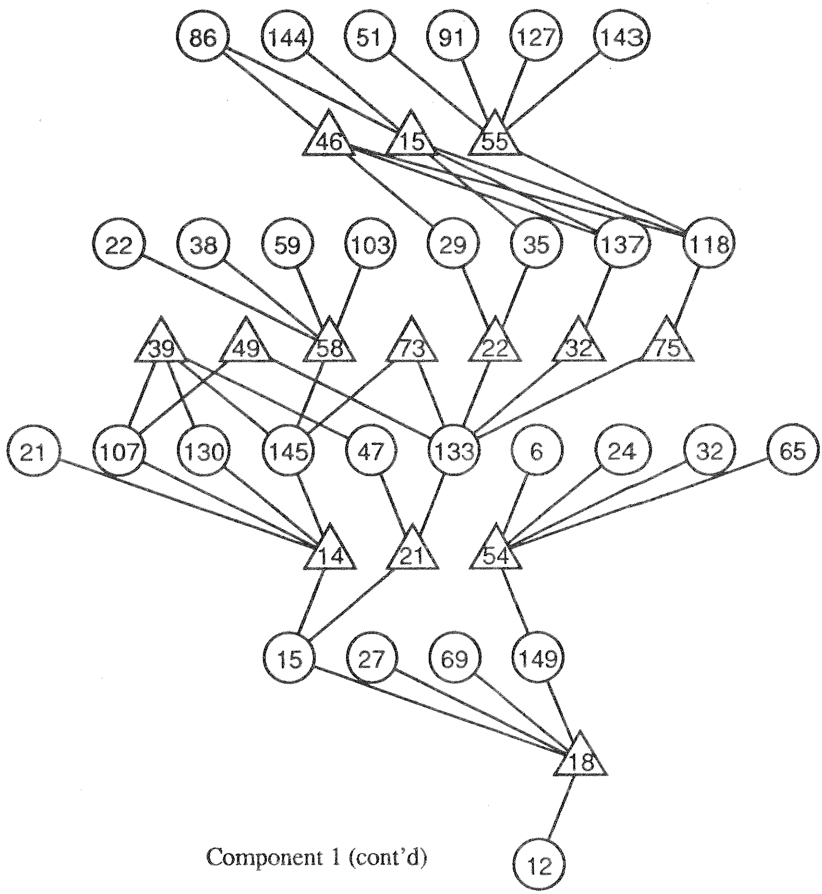
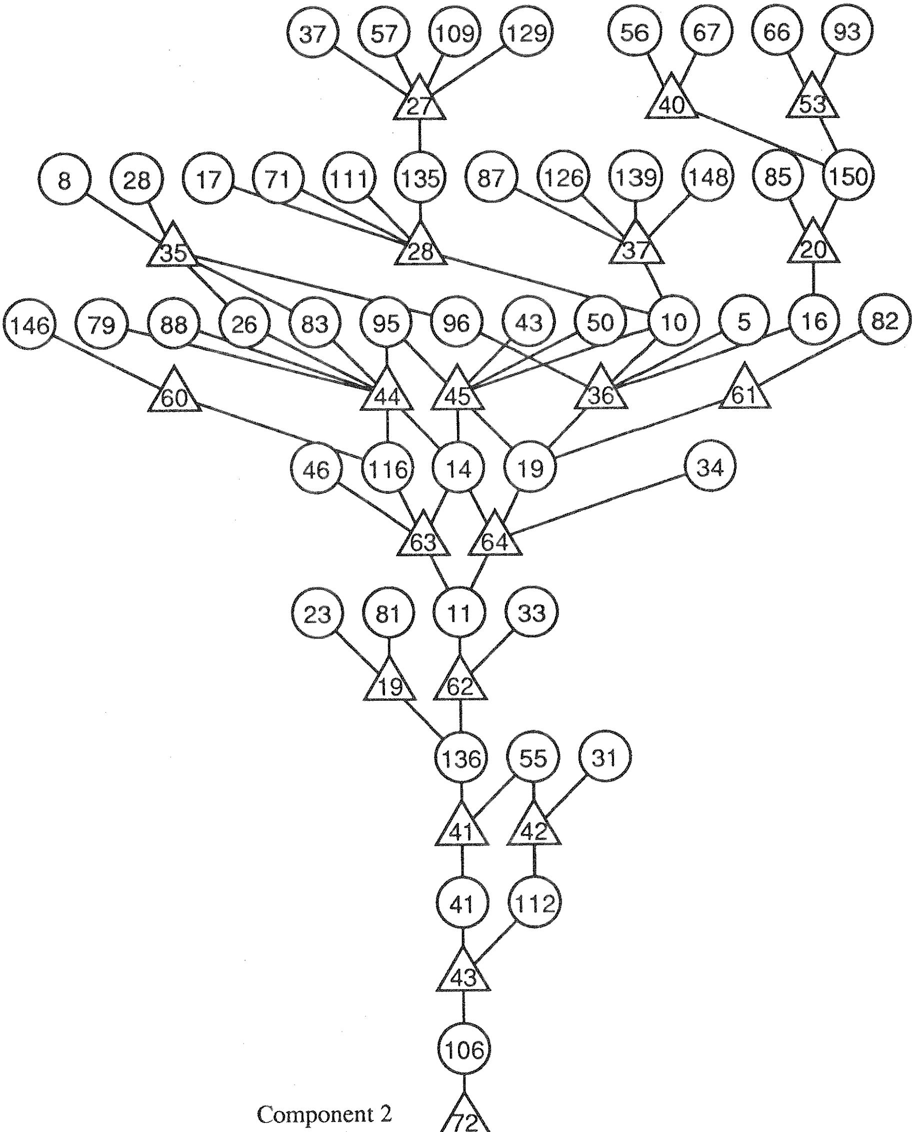


Figure A1: The relationships between near one–factorizations of K_9 (denoted by circles) and overlarge sets of 2–(9,3,1) designs (denoted by triangles) in the first part of component 1. The next diagram shows the rest of this component with overlarge set of 2–(9,3,1) designs, number 18, as the connecting point.



Component 1 (cont'd)

Figure A1(cont'd): The relationships between near one–factorizations of K_9 (denoted by circles) and overlarge sets of 2–(9,3,1) designs (denoted by triangles) in the second part of component 1. The previous diagram shows the rest of this component with overlarge set of 2–(9,3,1) designs, number 18, as the connecting point.



Component 2

Figure A1(cont'd): The relationships between near one-factorizations of K_9 (denoted by circles) and overlarge sets of 2–(9,3,1) designs (denoted by triangles) in component 2.

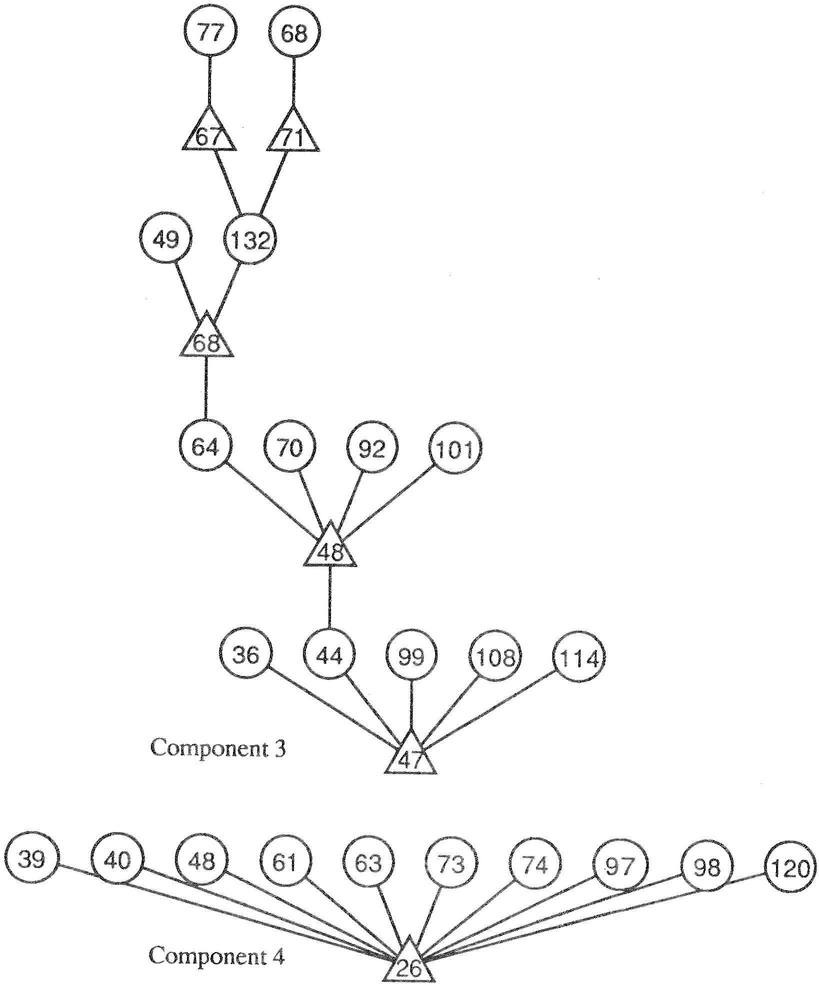


Figure A1(cont'd): The relationships between near one–factorizations of K_9 (denoted by circles) and overlarge sets of 2–(9,3,1) designs (denoted by triangles) in components 3 and 4.

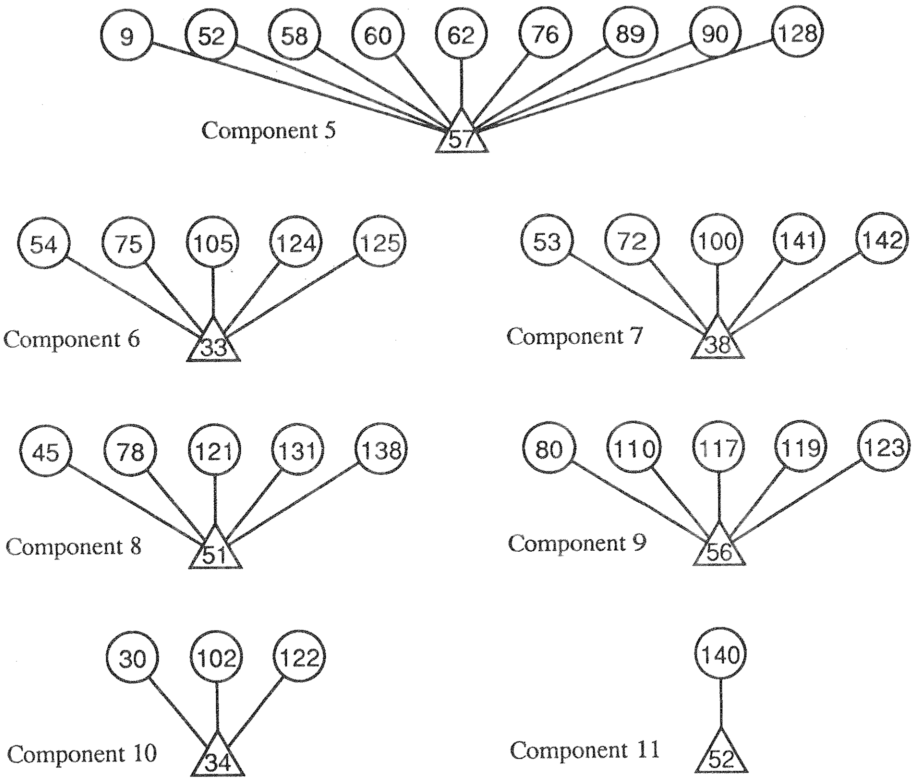


Figure A1(cont'd): The relationships between near one–factorizations of K_9 (denoted by circles) and overlarge sets of 2–(9,3,1) designs (denoted by triangles) in components 5 through 11.

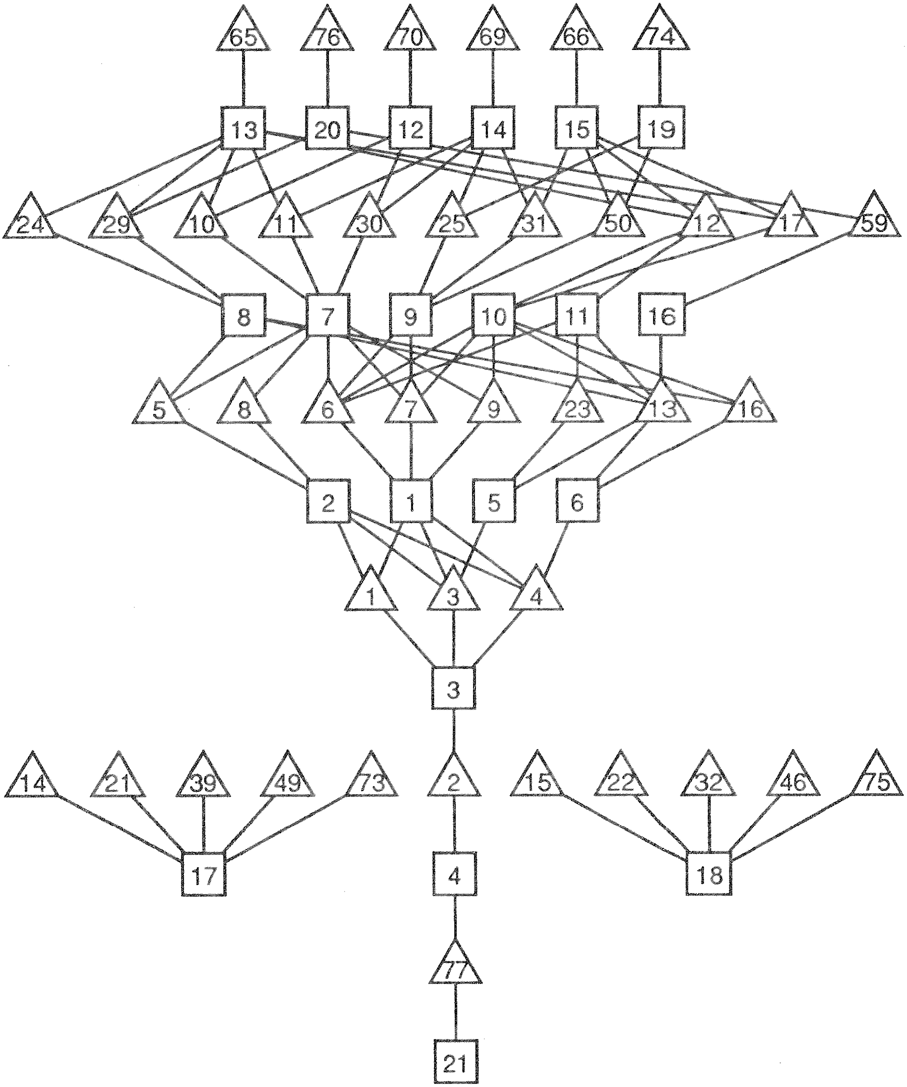


Figure A2: The relationships between overlarge sets of 2–(9,3,1) designs (denoted by triangles) and overlarge sets of 3–(10,4,1) designs (denoted by squares).

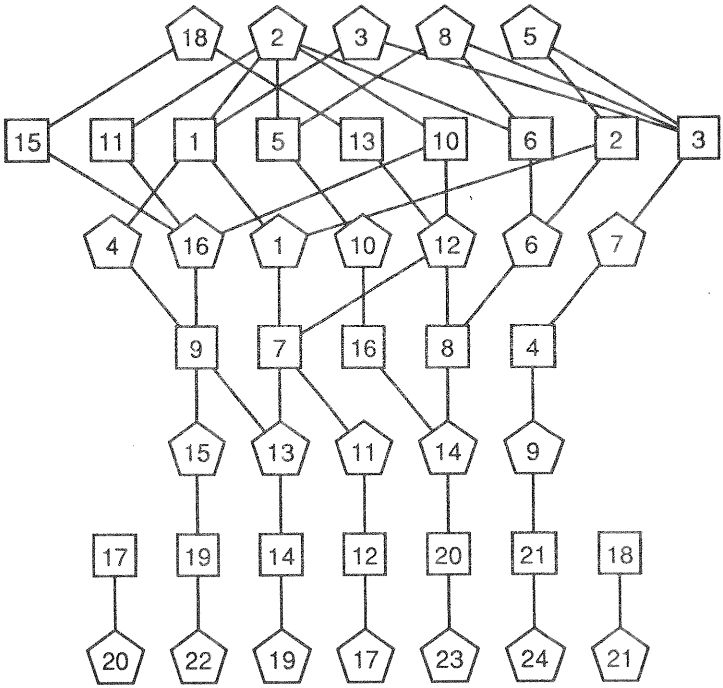


Figure A3: The relationships between overlarge sets of 3–(10,4,1) designs (denoted by squares) and overlarge sets of 4–(11,5,1) designs (denoted by pentagons).